(x,y) plane can, in general, be made by noting that

$$\xi = \int_0^\zeta \left(\frac{\mu}{\mu_w}\right) d\zeta \tag{16}$$

and substituting μ/μ_w as a function of ζ from its assumed variability with, e.g., T in conjunction with Eq. (15). Equations (11, 15, and 16) constitute an asymptotic suction solution for compressible flows with heat transfer for arbitrary $u_1(x)$, $v_w(x)$, $T_w(x)$, constant Pr, and arbitrary variable μ.

In view of the general conditions under which it is valid, the solution derived here is remarkably simple. First, in the special case of low-speed flows ($M_1 \approx 0$) with constant fluid properties† [but still variable $v_w(x)$ and $T_w(x)$], Eqs. (11) and (15) each reduce to a form quite similar to Eq. (1):

$$u/u_1(x) = 1 - e^{-\xi} \qquad \theta = 1 - e^{-Pr\xi} \tag{17}$$

where $\xi = \rho v_w(x)y/\mu$. Second, by setting $(\partial T/\partial\zeta)_w = 0$ and solving for T_w, Eq. (15) is found to imply that the equilibrium wall temperature T_e for zero heat transfer will be

$$T_e/T_1 = 1 + [(\gamma - 1)/2]M_1^2 \tag{18}$$

for all Prandtl numbers. Finally, it is noted that, in general, the local skin-friction coefficient and Nusselt number will be

$$c_f \equiv \frac{(\mu\partial u/\partial y)_w}{(\frac{1}{2})\rho_1 u_1^2} = 2\left(\frac{\rho_w v_w}{\rho_1 u_1}\right) \tag{19}$$

$$Nu \equiv \frac{(k\partial T/\partial y)_w x}{k_1(T_e - T_w)} = Pr\left(\frac{k_w}{k_1}\right)\left(\frac{\rho_w v_w x}{\mu_w}\right) \tag{20}$$

Thus, the Nusselt number here is directly proportional to Pr, in contrast to the case of an impermeable wall, for which Nu is a considerably more complicated function of Pr, depending somewhat on the pressure gradient and varying, for example, approximately as $Pr^{1/3}$ in a zero pressure gradient for $Pr \geq 0.6$. Equations (19) and (20) imply the following type of Reynolds analogy‡:

$$(Nu/c_f)R_x^{-1} = Pr/2 \tag{21}$$

The solutions obtained here may be regarded as approximate solutions of the laminar boundary layer for cases of *finite* but large suction velocities. From this point of view it may be of interest to compare these solutions with the similarity solutions recently obtained numerically by Koh and Hartnett[8] for low-speed flows with constant fluid properties, in which $u_1 \sim x^m$, $v_w(x) \sim x^{(m-1)/2}$, $T_w(x) - T_1 \sim x^n$, $Pr = 0.73$, and $m = 0, \frac{1}{3}, 1$; $n = -1$ to 10. In the notation of Ref. 8, Eqs. (17, 19, and 20) for these flows can be written as

$$u/u_1 = 1 - e^{-[(m+1)/2]f_w\eta} \qquad \theta = 1 - e^{-[Pr(m+1)/2]f_w\eta} \tag{22}$$

$$c_f R_x^{1/2} = (m+1)f_w \qquad Nu/R_x^{1/2} = Pr[(m+1)/2]f_w \tag{23}$$

A detailed comparison has shown that the velocity profiles of Eq. (22) virtually coincide with the profiles obtained in Ref. 8 for suction velocities corresponding to $f_w \geq 8$, while for $f_w = 6$ the difference throughout is within 3%. Moreover, for constant wall temperature ($n = 0$), the temperature profiles of Eq. (22) virtually coincide throughout with those of Ref. 8 for $f_w = 8$. For variable wall temperature, one readily can see from Figs. 3–5 of Ref. 8 the actual approach, with increasing suction, of the temperature profiles to the asymptotic suction profile by comparing the set of profiles for $n = -1$ to 10, corresponding to $f_w = 0$ and $f_w = 1$, with the set corresponding to $f_w = 8$. Whereas the effect of n on the temperature profiles is quite considerable in the case of zero suction ($f_w = 0$), the profiles for $n = -1$ to 10 are seen to come relatively close together (though not yet quite coincident), i.e., they tend to become independent of n, when $f_w = 8$.§

† That is, constant ρ, c_p, μ, and k. The temperature T, however, is permitted to vary.

‡ The relation $k_w/\mu_w = k_1/\mu_1$, following from the assumed constancy of Pr and c_p, is used here.

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§ An approximate solution for large suction, showing analytically the approach of the temperature profiles to the asymptotic profile for these flows, can be obtained by putting $f' = 1, f = f_w$ into Eq. (9) of Ref. 8 and solving for θ. The following solution thus is obtained for $n \geq 0$: $\theta = 1 - e^{-\alpha\eta}$, where

$$\alpha = \frac{Pr(m+1)}{4} f_w + \left[\left(\frac{Pr(m+1)}{4} f_w\right)^2 + nPr\right]^{1/2}$$

As $f_w \to \infty$, this solution approaches that in Eq. (22).

Effects of Controlled Roughness on Boundary-Layer Transition at a Mach Number of 6.0

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THE importance of the effects of boundary-layer transition on the aerodynamic characteristics and the heat transfer to a vehicle in flight at hypersonic speeds is well known. As had been discussed in Ref. 1, the considerable scatter found in the data currently available on boundary-layer transition may be due in part to the method of detecting transition. In this investigation of hypersonic boundary-layer transition on a smooth flat plate with a sharp leading edge (leading edge thickness less than 0.002 in.) and with or without controlled roughness, the heat flow rate method of detecting transition has been employed with considerable success. By quickly injecting the model from a sheltered position outside the tunnel wall into an established Mach 6.0 freestream, lateral conduction in the model skin has been kept to a minimum. The tunnel stagnation temperature

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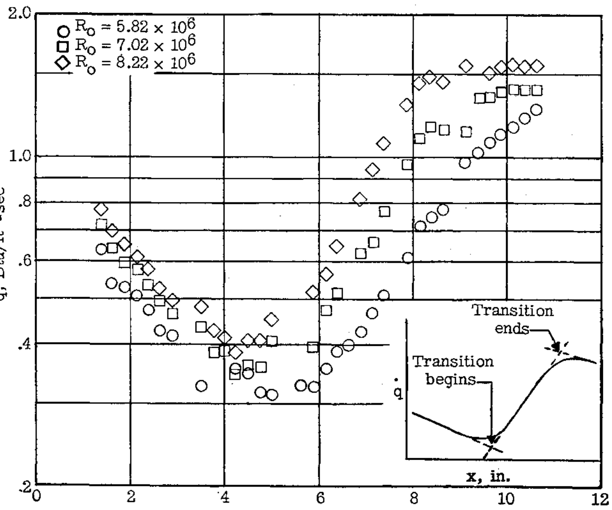

Fig. 1 Heating rate distribution on the continuous flat plate.

transition of the boundary layer. This unexpected phenomenon is thought to be a result of a laminar separation region in the proximity of the roughness elements. For $k = 0.0044$ ft, transition has moved considerably forward of the natural transition location; and for $k \geq 0.0054$ ft, fully developed turbulent flow begins in the region of the roughness elements. In this paper, the value of roughness height for which a further increase in height causes no appreciable forward movement of the beginning of fully developed turbulent flow is defined as the "critical" roughness height (hence, from Fig. 2, $k_{crit} = 0.0054$ for $R_0 = 8.22 \times 10^6$). Note that the critical roughness height found in Fig. 2 is more than double the calculated boundary-layer thickness at the roughness position.

was approximately 960°R, and the wall temperature of the plate at the time for which the experimental data are presented was approximately 550°R. In Fig. 1, the heat flow rates $\dot{q}$, along the centerline of the model without roughness are shown for several freestream unit Reynolds numbers per foot, R_0. The transition region is seen to be defined very well with the beginning and end of transition determined from the experimental data by the method illustrated in the insert of Fig. 1.

The heat flow rates for the model with various size roughness (obtained by interchangeable leading edge pieces) located at a longitudinal distance of 2.0 in. from the leading edge are presented in Fig. 2. Tabulated in the key of Fig. 2 are k, the vertical height of the spheres above the plate surface; d, the longitudinal spacing between the spheres; the roughness parameter, k/δ, where δ is the calculated boundary-layer thickness at the roughness location; and R_k, the Reynolds number based on flow conditions at the top of the roughness elements and the roughness height. From Fig. 2, it is evident that the addition of the smallest roughness elements ($k = 0.0018$ ft and 0.0030 ft) results in delaying the

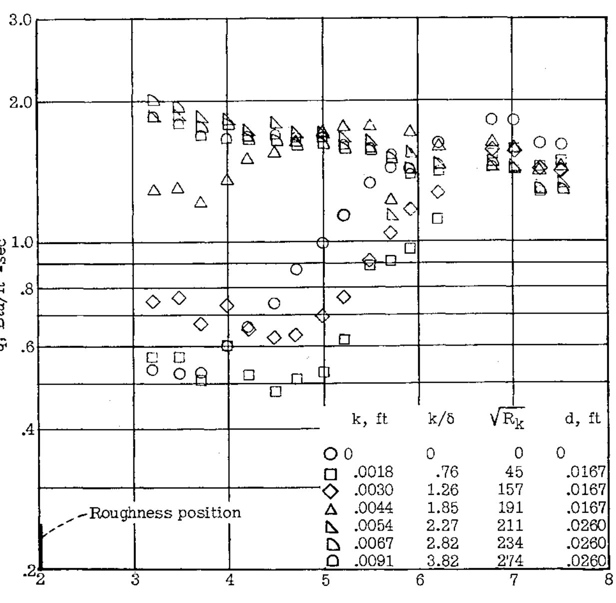

Fig. 2 Heating rate distribution along flat plate with various size roughness, $R_0 = 8.22 \times 10^6$.

The results of this investigation are summarized in Fig. 3 for local freestream Mach numbers of 6.0 and 4.8 (obtained with the model at an angle of attack of $-8°$). The Reynolds number range was obtained by varying the tunnel stagnation

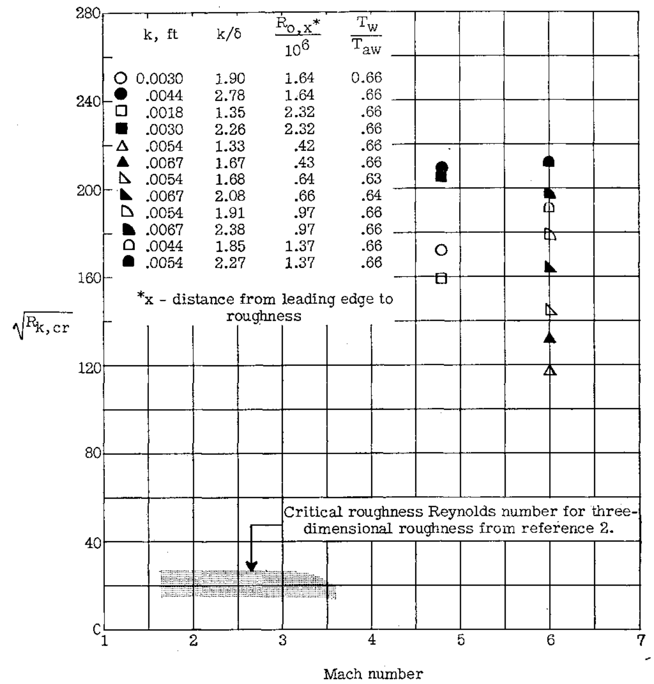

Fig. 3 Critical roughness Reynolds number at several Mach numbers. Open symbols indicate that the roughness height is slightly less than the critical value, and solid symbols indicate that the roughness height is slightly greater than the critical value.

pressure from approximately 165 to 515 psia. The experimental "critical" roughness Reynolds numbers are compared with those found in previous studies at lower supersonic Mach numbers.[2] The critical roughness Reynolds numbers have been found to increase sharply above the previously determined values in the lower supersonic range for the higher Mach numbers of this investigation. The critical roughness heights determined for Mach numbers of 4.8 and 6.0 are also much larger than those predicted from previous supersonic semi-empirical equations (e.g., the work of van Driest[3]). Hence, it has been shown experimentally that the roughness parameter, k/δ, required to move the beginning of fully developed turbulent flow approximately to the roughness position is much greater for higher supersonic and hypersonic Mach numbers than that previously established for low supersonic Mach numbers.

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Correlation of Laminar Heating to Cones in High-Speed Flight at Zero Angle of Attack

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Laminar heating to pointed cones in high-speed flight at zero angle of attack is correlated by a simple expression involving only freestream density, velocity, and cone angle. Comparison of the theory with experiments and a flat-plate laminar heating correlation is very satisfactory. It is found that the ratio of (dimensionless) wall enthalpy gradient parameters $(\partial g/\partial\eta)_w$ for cones and axisymmetric stagnation points is very nearly equal to unity.

Nomenclature

g = enthalpy ratio, H/H_e
g' = wall enthalpy gradient, $\partial g/\partial\eta$
H = total enthalpy
h = static enthalpy
p = static pressure
Pr = Prandtl number
q = heating rate, Btu-sec^{-1}-ft^{-2}
Q = integrated heating rate, Btu-sec^{-1}
r = distance from axis normal to U_∞, ft
R = blunt-body nose radius, ft
T = temperature
U = velocity, ft-sec^{-1}
x = axial distance, ft
x' = distance along body surface, ft
δ = cone half-angle
μ = absolute viscosity, slugs-(ft-sec)$^{-1}$
ξ = transformation variable, slugs^2-ft^{-2}
ρ = density, slugs-ft^{-3}

Subscripts

c = cone; external to cone boundary layer
e = external to boundary layer
s = blunt-body stagnation point
sl = sea level
w = wall
∞ = freestream

Certain aspects of re-entry analysis require knowledge of aerodynamic heating, and so it is desirable to have a means of determining this quantity in a simple, accurate manner for various body shapes. Stagnation heating to axisymmetric bodies already has been correlated satisfactorily as[1]

$$q_sR^{1/2} = 867(\rho_\infty/\rho_{sl})^{1/2}(U_\infty/10^4)^{3.15} \times [(H_e - h_w)/(H_e - h_{w_{300^\circ\mathrm{K}}})] \quad (1)$$

which agrees with detailed calculations (and experiments[1]) to within $\pm$ 10% for all altitudes up to 250 kft and velocities between 7 and 25 kft/sec. It is the purpose herein to present a similar relation for laminar heating to cones in hypersonic flight at zero angle of attack.

In light of the existence of Eq. (1), heating to cones is known if the ratio of cone to stagnation point heating can be evaluated. To this end, consider the case of Lewis number equal to 1, whereby laminar heating is given as[2]

$$q = (r\rho\mu g')_w U_e H_e/Pr(2\xi)^{1/2} \quad (2)$$

Assuming constant wall properties, the independent variable ξ is[2]

$$\xi_s = (\rho_w\mu_w)_s(x')^4(dU_e/dx')_s/4 \quad (3)$$

in the stagnation point region of blunt axisymmetric bodies, and

$$\xi_c = (\rho_w\mu_w Ux^3)_c \tan^2\delta/3\cos\delta \quad (4)$$

for cones. If H_e and T_w are the same in both cases, and $\mu = \mu(T)$, then the ratio of cone to axisymmetric stagnation heating follows from Eqs. (2–4) as

$$\frac{q_c}{q_s} = \frac{3^{1/2}}{2} G \left[\frac{\cos\delta}{x_c}\left(\frac{\rho_c}{\rho_s}\right)_w \frac{U_c}{(dU_e/dx')_s}\right]^{1/2} \quad (5)$$

where G is the ratio of wall enthalpy gradient parameters $(g_c'/g_s')_w$.

The density ratio $(\rho_c/\rho_s)_w$ is equal to the pressure ratio p_c/p_s, since wall temperatures are the same. In accordance with the Newtonian flow approximation,

$$p/p_\infty = 1 + \gamma_\infty(M_\infty \sin\delta)^2$$

where $\delta = \pi/2$ for a stagnation point. Therefore,

$$(\rho_c/\rho_s)_w = (p_c/p_\infty)/(p_s/p_\infty) \simeq 1/\gamma_\infty M_\infty^2 + \sin^2\delta \simeq \sin^2\delta \quad (6)$$

for $M_\infty \simeq 20$ and $\delta > 5°$.

The velocity external to the cone boundary layer U_c is given by the hypersonic approximation

$$U_c = U_\infty \cos\delta \quad (7)$$

Combining Eqs. (5–7) and using the velocity derivative $(dU_e/dx')_s$ corresponding to Newtonian flow in the stagnation region,[3] one obtains

$$q_c\, x_c^{1/2}/q_s\, R^{1/2} = 0.364\, G \sin 2\delta(\rho_s/\rho_\infty)^{1/4} \quad (8)$$

as the cone-stagnation point heating ratio.

The stagnation density ratio ρ_s/ρ_∞ is given by Feldman[4] over the velocity range $8 \leq U_\infty \leq 24$ kft/sec for altitudes up to 250 kft. His data are correlated by

$$\rho_s/\rho_\infty = 8.4(U_\infty/10^4)^{0.8} \quad (9)$$

the one-fourth power of which is accurate to within 6% for all velocities within this range and altitudes greater than 50 kft.

Therefore, inserting Eq. (9) into (8) and using (1), heating to cones in high-speed flight at zero angle of attack is given as (leaving out the wall enthalpy correction)

$$q_c\, x_c^{1/2} = 535\, G \sin 2\delta(\rho_\infty/\rho_{sl})^{1/2}(U_\infty/10^4)^{3.35} \quad (10)$$

The total heat input Q follows by integrating Eq. (10) over the body surface (exclusive of the base):

$$Q = 4500\, GL^{3/2} \sin\delta \tan\delta(\rho_\infty/\rho_{sl})^{1/2}(U_\infty/10^4)^{3.35} \quad (11)$$

To complete the correlation, Eq. (10), it remains to specify the ratio of wall enthalpy gradient parameters G.

A result of the calculations of Kemp et al.[3] was that the wall enthalpy gradient parameter $g_w'/(1 - g_w)$ is practically unaffected by the level of the dissipation parameter U_e^2/H_e for a given value of pressure gradient parameter β. This means that the difference between g_{wc}' and g_{ws}' arises primarily because $\beta = 0$ for a cone and $\frac{1}{2}$ for a (spherical) stag-

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